5

Stereographic Projection

Objective

- **Use stereographic projection to quantitatively represent three-dimensional, orientation data (such as the attitudes of lines and planes) on a two-dimensional piece of paper.**

An extremely useful technique for solving many structural problems is stereographic projection. This involves the plotting of planes and lines on a circular grid or net. Many computer programs are available, both commercially and as freeware, that will plot and manipulate structural data on stereonets in different and powerful ways. However, in this chapter we introduce you to some of these techniques and encourage you to complete the problems in the traditional way—by hand. Plotting and manipulating structural data by hand forces you to visualize the connection between the orientations of structures in the field or on a map and the sometimes complex patterns of lines plotted on the stereonet. Your ability to correctly interpret stereograms, regardless of how they are generated, will be severely limited if you are not adept at plotting and manipulating structural data by hand.

Two types of nets are in common use in geology. The net in Fig. 5.1a is called a stereographic net; it is also called a Wulff net, after G. V. Wulff, who adapted the net to crystallographic use. The net in Fig. 5.1b is called a Lambert equal-area net, or Schmidt net. In common geologic parlance, both of these nets are referred to as stereonets.

The two nets are constructed somewhat differently. On the equal-area net, equal areas on the reference sphere remain equal on the projection. This is not the case with the stereographic net. The situation is similar to map projections of the earth; some projections sacrifice accuracy of area to preserve spatial relationships, while others do the opposite. In preserving area, the equal-area net does not preserve angular relationships. The construction of the equal-area net does allow the correct measurement of angles, however, and this net may reliably be used even when angular relationships are involved. In structural geology the relative density of data points is often important, so most structural geologists use the equal-area net. In crystallography, angular relationships are especially important, so crystallographers use the Wulff net. In this book, we will use the equal-area net exclusively.

The equal-area net is arranged rather like a globe of the earth, with north–south lines that are analogous to meridians of longitude and east–west lines that are analogous to parallels of latitude. The north–south lines are called *great circles* and the east–west lines are called *small circles*. The perimeter of the net is called the *primitive circle* (Fig. 5.2);

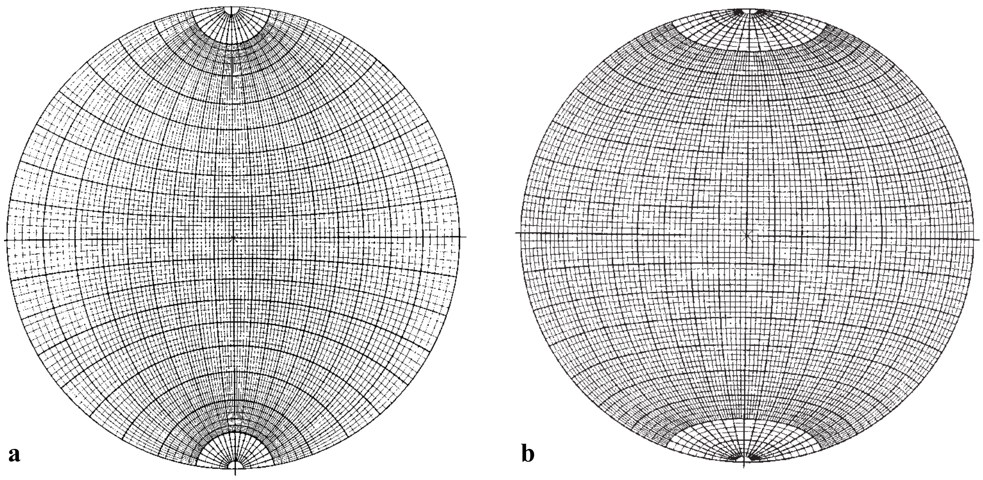

Fig. 5.1 Nets used for stereographic projection. (a) Stereographic net or Wulff net. (b) Lambert equal-area net or Schmidt net.

here "primitive" has the mathematical sense of "fundamental."

Unlike crystallographers, who use the net as if it were an upper hemisphere, structural geologists use it as a lower hemisphere. To visualize how elements are projected onto the net, imagine looking down into a large bowl in which a cardboard half-circle has been snugly fitted at an angle. The exposed diameter of the half-circle is a straight line, and the curved part of the half-circle describes a curve on the bottom of the bowl. Figure 5.3a is an oblique view of a plane within a bowl; the plane strikes north–south and dips 50°W. Figure 5.3b is an equal-area projection of the same plane. Notice that the dip of the plane, 50° in this case, is measured from the perimeter of the net. The great circles on the net represent a set of planes having the same strike and all possible dips. The primitive circle represents a horizontal plane. Although north and south poles are labeled in Figs 5.2 and 5.3, geographic coordinates are actually attached to the data on the tracing paper that you will lay over your net, rather than to the underlying net. By rotating the tracing paper, a great circle corresponding to any plane may be drawn. Similarly, we will refer to the straight line that corresponds to the equator in Figs 5.2 and 5.3 as the "east–west line," even though it has no fixed geographic orientation.

Figure G-11 (Appendix G) is an equal-area net for use in this and succeeding chapters. You will project lines and planes onto the net by placing a piece of tracing paper over the net and rotating the tracing paper on a thumbtack located at the center of the net. Your net will be heavily used, so it is a good idea to tape it to a piece of thin cardboard to protect it and to ensure that the thumbtack hole does not get larger. Place a rubber eraser on the thumbtack when your net is not in use, to avoid impaling yourself.

Following are several examples of stereographic projection. Work through each of them on your own net.

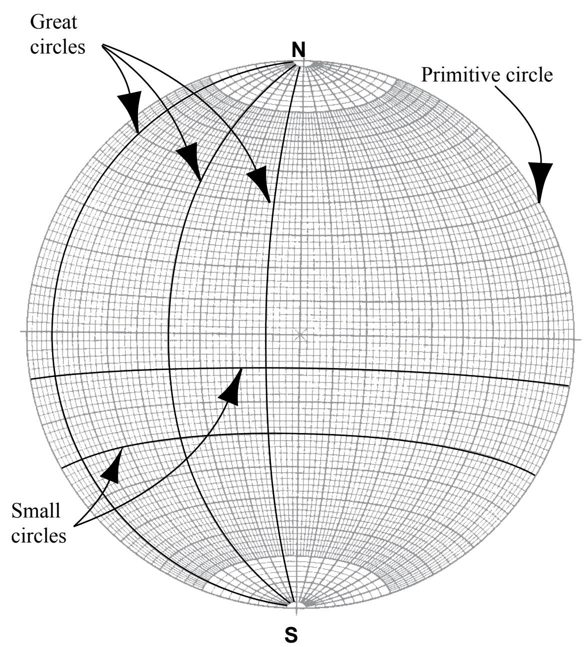

Fig. 5.2 Main elements of the equal-area projection.

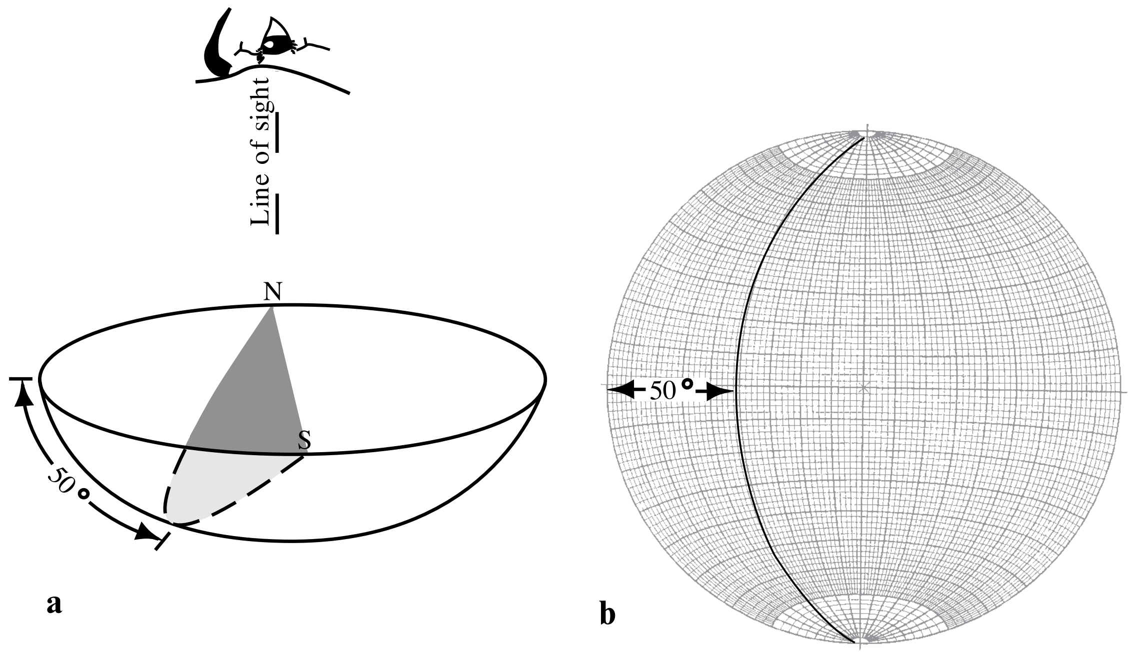

Fig. 5.3 Oblique lower-hemisphere view of the projection of a plane striking north–south and dipping 50° west. (a) Oblique view. (b) Equal-area projection.

A plane

Suppose a plane has an attitude of 315°, 60°SW. It is plotted on the equal-area net as follows:

1 Stick a thumbtack through the center of the net from the back, and place a piece of tracing paper over the net such that the tracing paper will rotate on the thumbtack. A small piece of clear tape in the center of the paper will prevent the hole from getting larger with use.
2 Trace the primitive circle on the tracing paper (this step may be eliminated later), and mark the north and south poles.
3 Find 315° on the primitive circle, mark it with a small tick mark on the tracing paper and label it 315° (Fig. 5.4a).
4 Rotate the tracing paper so that the N45°W mark is at the north pole of the net (Fig. 5.4b).
5 Southwest is now on the left-hand side of the tracing paper, so count 60° inward from the primitive circle along the east–west line of the net and put a mark on that point.
6 Without rotating the tracing paper, draw the great circle that passes through that point (Fig. 5.4b).
7 Finally, rotate the paper back to its original position (Fig. 5.4c).

A line

While a plane intersects the hemisphere as a line, a line intersects the hemisphere as a single point. Figure 5.5a is an oblique view of a line that trends due west and plunges 30°, and Fig. 5.5b is an equal-area net projection of this line. Imagine this line passing through the center of the sphere and piercing the lower hemisphere.

Consider a line that has an attitude of 32°, S20°E. Here is how such a line projects onto the net:

1 Mark the north pole and S20°E on the tracing paper.
2 Rotate the S20°E point to the bottom ("south") point on the net. (The north, west, or east points work just as well. Only from one of these four points on the net may the plunge of a line be measured.)
3 Count 32° from the primitive circle inward, and mark that point (Fig. 5.6).
4 Rotate the paper back to its original orientation.

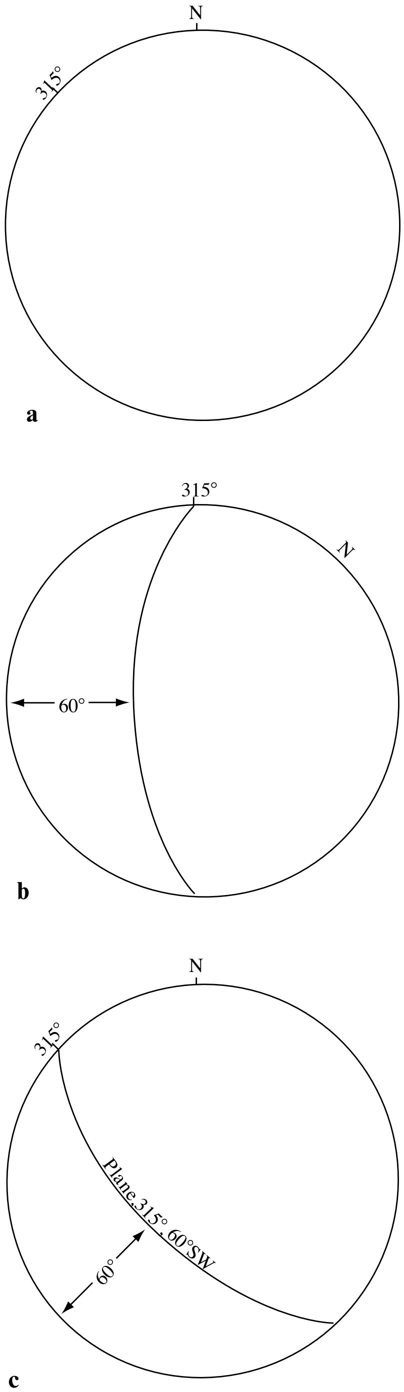

Fig. 5.4 Projection of a plane striking 315° and dipping 60°SW. (a) Plotting of strike. (b) Projection of plane with tracing paper rotated so that the strike is at the top of the net. (c) Tracing paper rotated back to original position.

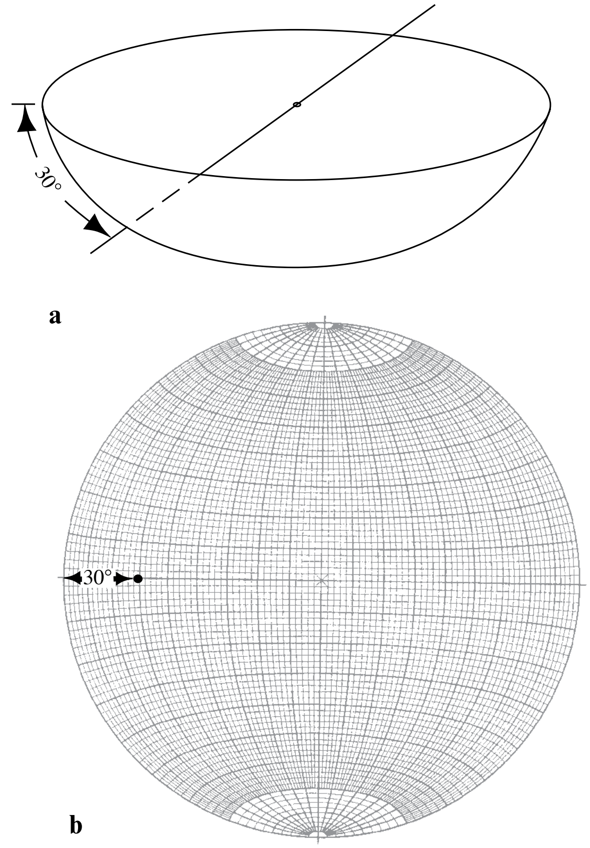

Fig. 5.5 Projection of a line that plunges 30° due west. (a) Oblique view. (b) Equal-area projection.

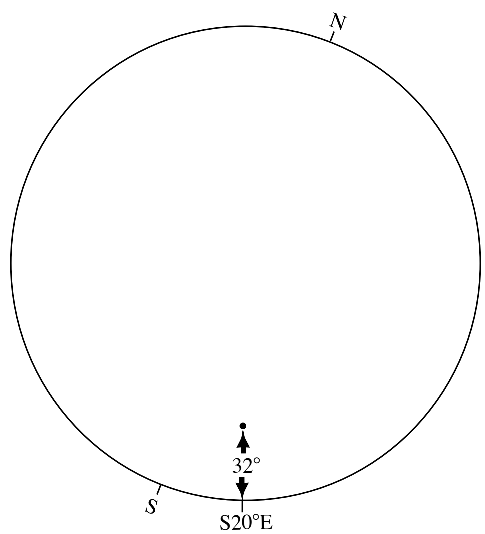

Fig. 5.6 Projection of a line that plunges 32°, S20°E.

Pole of a plane

By plotting the pole to a plane it is possible to describe the plane's orientation with a single point on the net. The pole to a plane is the straight line perpendicular to the plane, 90° across the stereonet from the great circle that represents the plane. If a plane strikes north–south and dips 40° west, its pole plunges 50° due east (Fig. 5.7).

Suppose a plane has an attitude of N74°E, 80°N. Its pole is plotted as follows:

1 Mark the point on the tracing paper that corresponds to N74°E. Then rotate the tracing paper so that this point lies at the north pole of the net, as if you were going to plot the plane itself. The great circle representing this plane is shown by the dashed line in Fig. 5.8.
2 Find the point on the east–west line of the net where the great circle for this plane passes, and count 90° in a straight line across the net. This point is the pole to the plane. Mark this point, record the plunge, and also make a tick mark where the east–west line meets the primitive circle. Then rotate the tracing paper back to its original position and determine the direction of plunge. As shown in Fig. 5.8, the pole to this plane plunges 10°, S16°E.

In this way the orientation of numerous planes may be displayed on one diagram without cluttering it up with a lot of lines.

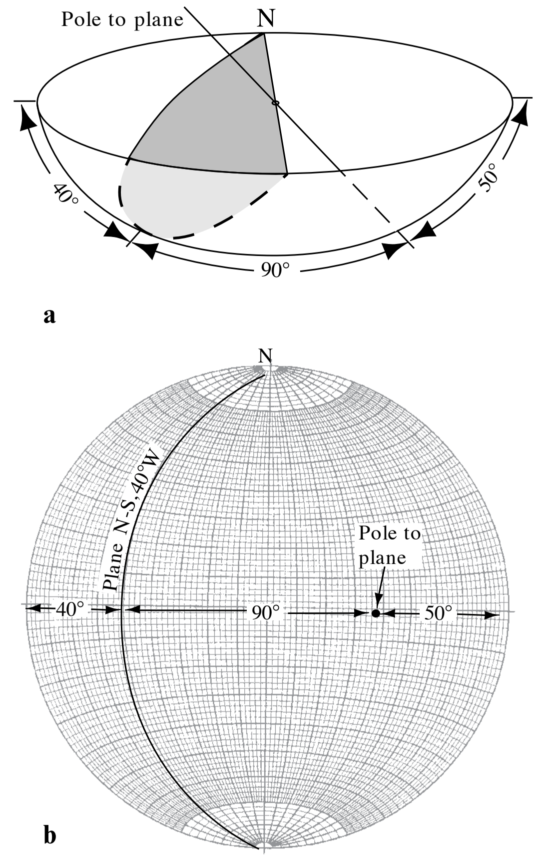

Fig. 5.7 Projection of a plane (N–S, 40°W) and the pole to the plane. (a) Oblique view. (b) Equal-area projection.

Line of intersection of two planes

Many structural problems involve finding the orientation of a line common to two intersecting planes. Suppose we wish to find the line of intersection of a plane 322°, 65°SW with another plane 060°, 78°NW. This line is located as follows:

1 Draw the great circle for each plane (Fig. 5.9).
2 Rotate the tracing paper so that the point of intersection lies on the east–west line of the net. Mark the primitive circle at the closest end of the east–west line.
3 Before rotating the tracing paper back, count the number of degrees on the east–west line from the primitive circle to the point of intersection; this is the plunge of the line of intersection.

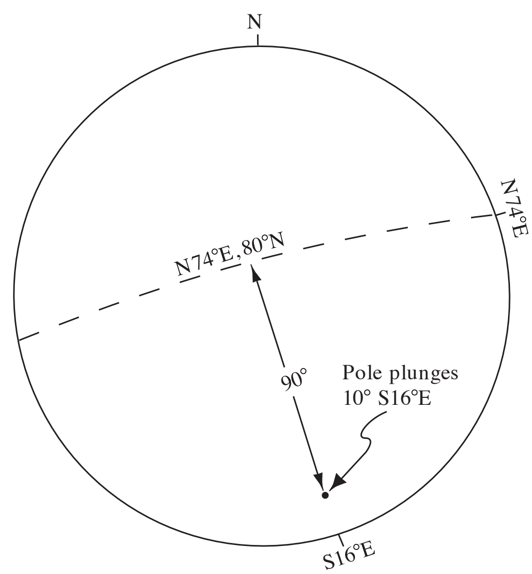

Fig. 5.8 Projection of a pole to a plane.

4 Rotate the tracing paper back to its original orientation. Find the bearing of the mark made on the primitive circle in step 2. This is the trend of the line of intersection. The line of intersection for this example plunges 61°, 264°, as seen in Fig. 5.9.

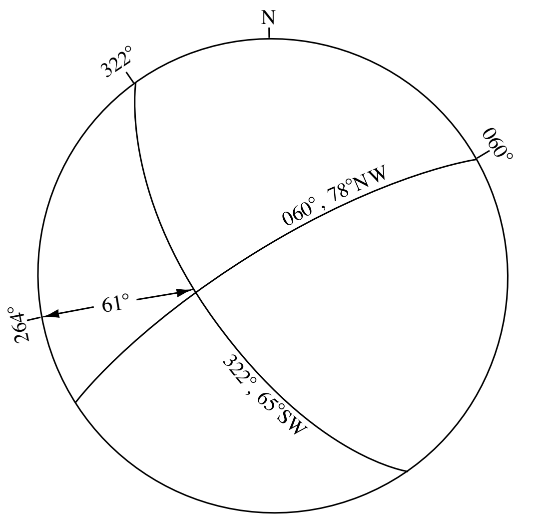

Fig. 5.9 Projection of the line of intersection of two planes. The attitude of the line of intersection is indicated.

Angles within a plane

Angles within a plane are measured along the great circle of the plane. In Fig. 5.10, for example, each of the two points represents a line in a plane that strikes north–south and dips 50°E. The angle between these two lines is 40°, measured directly along the plane's great circle.

The more common need is to plot the pitch of a line within a plane. Plotting pitches may be useful when working with rocks containing lineations. The lineations must be measured in whatever outcrop plane (e.g., foliation or fault surface) they occur. Suppose, for example, that a fault surface of N52°W, 20°NE contains a slickenside lineation with a pitch of 43° to the east (Fig. 5.11a). Figure 5.11b shows the lineation plotted on the equal-area net.

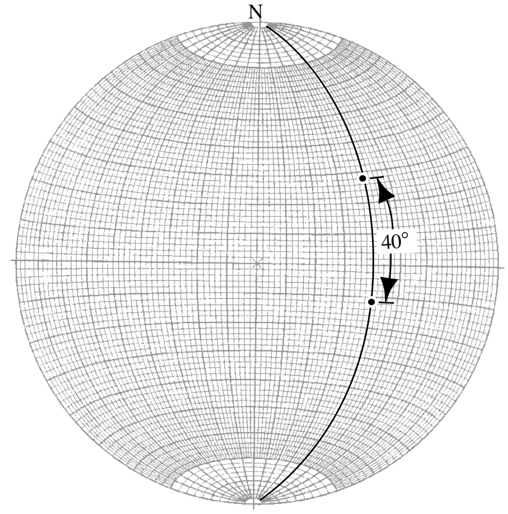

Fig. 5.10 Measuring the angle between two lines in a plane.

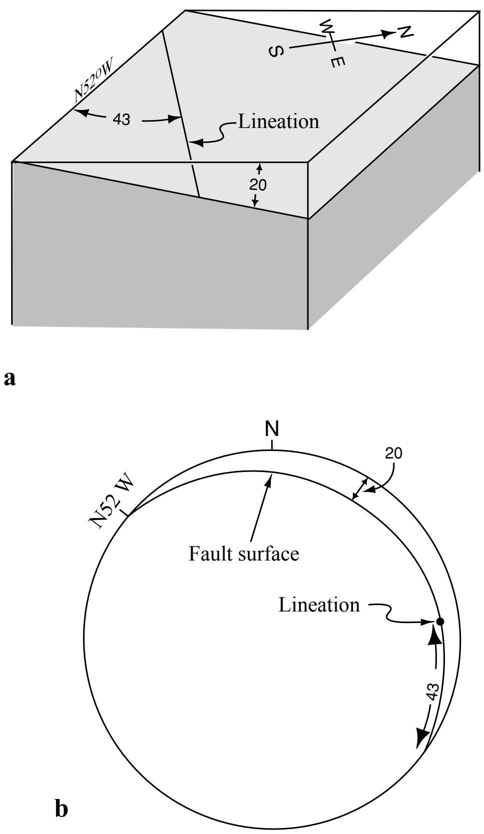

Fig. 5.11 Pitch of a line in a plane. (a) Block diagram. (b) Equal-area projection.

True dip from strike and apparent dip

Two intersecting lines define a plane, so if the strike of a plane is known, along with the trend and plunge of an apparent dip, then these two lines may be used to determine the complete orientation of the plane.

Suppose a fault is known to strike 010°, and an apparent dip of the fault plane is measured to have a trend of 154° and a plunge of 35°. The true dip of the fault is determined as follows:

1 Draw a line representing the strike line of the plane. This will be a straight line across the center of the net, intersecting the primitive circle at the strike bearing (Fig. 5.12a).
2 Make a pencil mark on the primitive circle representing the trend of the apparent dip (Fig. 5.12a).
3 Rotate the tracing paper so that the point marking the apparent-dip trend lies on the east–west line of the net. Count the number of degrees of plunge toward the center of the net and mark that point. This point represents the apparent-dip line.
4 We now have two points on the primitive circle (the two ends of the strike line) and one point not on the primitive circle (the apparent-dip point), all three of which lie on the fault plane. Turn the tracing paper so that the strike line lies on the north–south line of the net, and draw the great circle that passes through these three points.
5 Before rotating the tracing paper back, measure the true dip along the east–west line of the net. As shown in Fig. 5.12b, the true dip is 50°.

Strike and dip from two apparent dips

Even if the strike of a plane is not known, two apparent dips are sufficient to find the complete attitude. Suppose two apparent dips of a bed are 13°, S18°E and 19°, S52°W. The attitude of the plane may be determined as follows:

1 Plot points representing the two apparent-dip lines (Fig. 5.13a).
2 Rotate the tracing paper until both points lie on the same great circle. This great circle represents the plane of the bed, and the strike and dip are thus revealed. As shown in Fig. 5.13b, the attitude of the plane in this problem is N57°W, 20°SW.

Use stereographic projection to solve the following problems, using a separate piece of tracing paper for each problem.

Problem 5.1

Along a vertical railroad cut a bed has an apparent dip of 20°, 298°. The bed strikes 067°. What is the true dip?

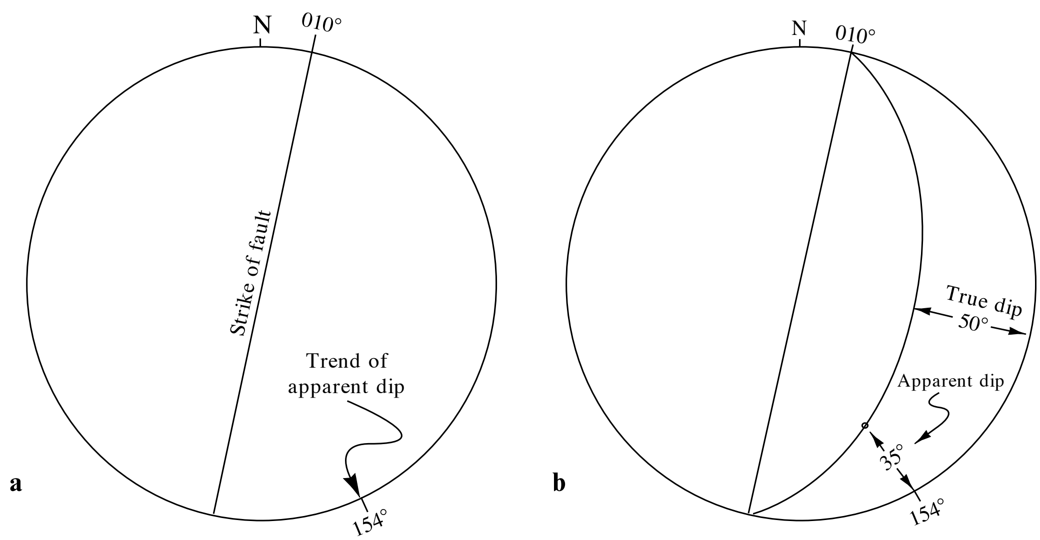

Fig. 5.12 Determination of true dip from strike and apparent dip. (a) Draw the strike line and trend of apparent dip. (b) Completed diagram showing the direction and degrees of the true dip.

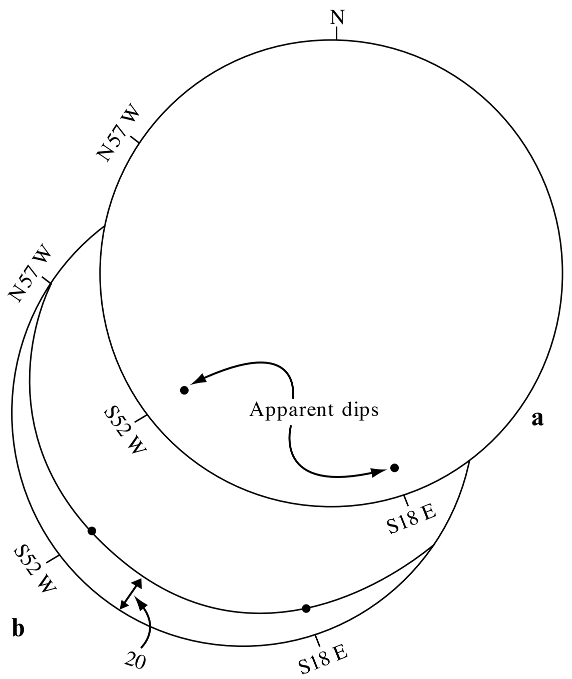

Fig. 5.13 Determination of strike and dip from two apparent dips. (a) Apparent dips plotted as points on the net. (b) Completed diagram showing the strike and true dip.

Problem 5.2

In a mine, a fault has an apparent dip of 14°, N90°W in one adit and 25°, S11°E in another. What is the attitude of the fault plane?

Problem 5.3

A fault strikes due north and dips 70°E. A limestone bed with an attitude of 325°, 25°SW is cut by the fault. Hydrothermal alteration along the fault has resulted in an ore shoot at the intersection of the two planes.

1 What is the orientation of the ore shoot?
2 What is the pitch of the ore shoot in the plane of the fault?
3 What is the pitch of the ore shoot in the plane of the limestone bed?

Problem 5.4

One limb of a fold has an attitude of 061°, 48°SE and other limb 028°, 55°NW. What is the orientation of the fold axis?

Problem 5.5

A coal bed with an attitude of N68°E, 40°S is exposed near the bottom of a hill. One adit is to be driven westward along the bottom of the bed from the east side of the hill, and a second adit is to be driven eastward along the bottom of the bed from the west side of the hill. To facilitate drainage of water, as well as the ease of movement of full ore carts out of the mine, each adit is to have a slope of 10°. Determine the bearing of each of the two adits.

Problem 5.6

You are mapping metamorphic rocks and you notice a lineation within the rocks. At five different outcrops you measure the pitch of the lineation on an exposed planar rock surface (not necessarily the same structural surface at each outcrop). The table below lists the attitude of each exposed surface and the pitch of the lineation on that surface.

Outcrop no.	Attitude of surface	Pitch of lineation
1	300°, 84°NE	76°E
2	350°, 30°E	50°N
3	040°, 70°SE	63°SW
4	337°, 30°W	50°S
5	272°, 45°N	59°E

You hypothesize that this lineation represents a planar fabric within the rock. Test this idea by determining whether these five lineation orientations are coplanar. If so, what is the attitude of the plane?

Problem 5.7

Two intersecting shear zones have the following attitudes: N80°E, 75°S and N60°E, 52°NW.

1 What is the orientation of the line of intersection?
2 What is the orientation of the plane perpendicular to the line of intersection?
3 What is the obtuse angle between the shear zones within this plane?
4 What is the orientation of the plane that bisects the obtuse angle?
5 A mining adit is to be driven to the line of intersection of the two shear zones. For maximum stability the adit is to bisect the obtuse angle between the two shear zones and intersect the line of intersection perpendicularly. What should the trend and plunge of the adit be?
6 If the adit is to approach the line of intersection from the southeast, will the full ore carts be going uphill or downhill as they come out of the mine?